Gimmel: Second Order Effect of Dynamic Policyholder Behavior on Insurance Products with Embedded Options

By John J. Wiesner, Charles L. Gilbert and David L. Ross

THE GLOBAL FINANCIAL CRISIS THAT STARTED IN 2008 highlighted the importance of higher order and cross Greeks in dynamic hedging programs used by insurance companies to manage risks associated with products such as variable annuities that provide investment guarantees. These guarantees represent embedded derivatives in the liabilities that are often complex, path dependent options. As such, sophisticated models are required to value the option and measure the sensitivity of this value to changes in the underlying,[1] yield curve, and volatility surface as well as the effect of the passage of time.

In general, the first order Greeks that measure the sensitivity to these financial variables (i.e., delta, partial rho, and partial vega) along with the passage of time (i.e., theta), capture most of the change in the option value when volatility is low. During times of higher volatility, second order Greeks such as Gamma, Vomma and Rho Convexity become more important. Following the financial crisis of 2008, more attention is also being given to third order and cross Greeks such as Speed, Ultima, and Vanna.

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Another important consideration for insurance companies is the effect that policyholder behavior will have on lapse rates and the resulting impact this will have on the value of the option. This paper defines a new measure, *Gimmel*, which captures the sensitivity of dynamic policyholder behavior (DPB) on the option value. As more experience data on policyholder behavior becomes available, dynamic policyholder behavior can be better defined as a function of the underlying. This then provides a way to measure the impact on the second order sensitivity, Gamma, to a change in underlying due to dynamic policyholder lapses. This is important because it reflects the fact that the embedded derivative in a variable annuity contract is in effect a put option on a put option.

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Dynamic hedging programs that have been established to manage the risks associated with equity-based guarantees are receiving greater attention. The financial crisis has highlighted that the risks within liabilities with complex guarantees is far more volatile and difficult to hedge than was previously thought. There is growing recognition of the importance of policyholder behavior within the insurance industry. Actuarial bodies are collecting experience data on policyholder behavior and quantifying the impact on the cost of investment guarantees associated with variable annuities and segregated funds.

The growing awareness of these issues and market turbulence has resulted in greater focus on the hedge effectiveness and the risk distribution of the hedging cost. The level of sophistication of dynamic hedging programs and stochastic modeling capabilities of insurers has increased significantly in just the last few years. While many insurers still execute first order dynamic hedging strategies (mostly hedging Delta and Rho), an increasing number are executing or evaluating second and higher order dynamic hedging strategies (including Vega and Gamma as well as third order and cross Greeks). Gamma, when not hedged by actual options, is sometimes hedged by variance swaps. Cross greeks such as delta's sensitivity to volatility may be partially hedged by VIX options. Gamma, third order and cross greeks may also be hedged by complex portfolios of options with multiple strikes and multiple expiries that may or may not actually match the underlying liabilities. It is not the focus of this paper to explain all the various strategies for hedging these greeks, but to highlight the increased sophistication of both the study and management of these complex liabilities.

Many of the models used for simulating stock prices would assume the large movements that occurred in the financial markets to be five standard deviations or higher

FOOTNOTES:

[1] In this paper we will assume for convenience that the underlying is an equity index.

> "Just as Gamma changes the Delta based on in-the-money-ness or out-of-the-money-ness, so likewise would rational policyholder behavior."

events which would not generally be considered in hedging programs. This level of volatility would significantly increase the hedging cost of first order dynamic hedging strategies and severely punish any insurer with a naked short Gamma position.

Not unlike Gamma, there is another factor that can significantly change the Delta of a liability with embedded guarantees—DPB. Just as Gamma changes the Delta based on in-the-money-ness or out-of-the-money-ness, so likewise would rational policyholder behavior (where we define rational[2] to be a policyholder who understands the value of the embedded guarantees within his or her policy). The further in-the-money (ITM) an option is, the closer the Delta gets to one. Similarly, the more in-the-money a guarantee gets, the less likely a *rational* policyholder will lapse.

Conversely, the further out-of-the-money an option gets, the closer the Delta of that option gets to zero, and the further out-of-the-money a guarantee gets the more likely the *rational* policyholder will lapse.[3]

Generally, an insurance policy with a guarantee is considered to be an option and modeled as such. In reality, the fact that the policyholders can lapse their policy means that the policy could also be considered as a consecutive series of options on an option. Each year, the policyholder can choose to continue owning the main option or choose to lapse the policy; the policyholder has the *option* of dropping the policy. Many policies have early termination penalties[4] to recapture some of the embedded value that these secondary options give the policyholders.

If these series of options were utilized by policyholders in a completely rational manner, the effect could be devastating to insurance companies and reinsurers. This stream of options on the main option has the effect of magnifying or compounding the Gamma effect of the original option in the guarantee. It is this effect—this further increase in the negative convexity of the guarantee beyond Gamma—that we have dubbed "Gimmel."

It might be important to distinguish a generic lapse assumption from the dynamic lapse assumption at this point. A generic non-dynamic lapse assumption tends to decrease the liability to the insurance company (i.e., it is beneficial to the company when a policyholder lapses). These kinds of products are known to be "lapse supported," in other words, lapses generally help the insurer by eliminating an obligation that the insurer had had. This sensitivity of the value of the liability with regard to flat-out lapses is quite different than the sensitivity to the *rational utility* of the policyholders. If those very same assumed lapses were to happen ONLY when the guarantee was not in the policyholders

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FOOTNOTES:

[2] This definition of "rational" does not include the possibility of liquidity and opportunity issues that may in fact make lapsing a policy and foregoing the embedded value of the guarantee a "rational" decision. As the secondary market for insurance products grows, insurers should be aware of the risk that lapses that would have been "rational" from a liquidity perspective may be curtailed as the secondary market provides liquidity to the policyholder without the policyholder necessarily needing to lapse the policy. Again, this paper is not intended to provide the "right" definition of "rational," but rather to provide a language that can help discussions of changing experience over time. This paper and its example focus purely on the economic value of the guarantee compared to the economic value of replacing the guarantee with separate option trades.

[3] Some policies have ratchets built in to minimize how far out-of-the-money OTM the guarantee will get precisely in order to discourage lapses. These ratchets though have an optionality value themselves that must be considered.

[4] Well designed early termination penalties should help decrease "short Gamma" on two counts; first by extending the expected duration of the overall "option", Gamma will be decreased as long dated options have less Gamma than shorted dated options, ceteris paribus; and second, the "options on the options" are less likely to be optimally utilized since there is an immediately recognizable cost to lapsing, thereby decreasing "Gimmel" itself. As these two effects will be taking place simultaneously, it may be difficult to separate the two effects. Ideally, a termination provision would encourage lapses when the guarantee is in the money, and discourage lapses when it is OTM.

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advantage, the result of the lapses would be quite detrimental to the insurer, rather than helpful.

However a modeler arrives at the cost of the *rational utility*, and whatever name is given for that cost, that is still not the sensitivity that is "Gimmel." Gimmel rather, is the change in the sensitivity of the value of the liability to changes in the underlying funds.

As an unparameterized definition of this sensitivity of the liability we offer:

Gimmel (𐤂) the change in the Delta of a investment with a guarantee with regard to a change in the underlying due to Dynamic Policyholder Behavior; or, more simply, the incremental change in Gamma due to Dynamic Policyholder Behavior.[5]

Let,
ω = total lapses
$\omega = \omega^b + \omega'$

Where,
ω^b = base lapses that do not vary with underlying
ω' = dynamic lapses in excess of base lapses that are a function of the underlying, in-the-moneyness, and degree of rationality (0% - 100%). Dynamic lapses could also be a function of the price of the option—i.e., vol, T-t, risk-free rate, etc.—and would make Gimmel a function of multiple financial variables, which it could very well be.

Then,

𐤂 $= \delta\Gamma/\delta\omega'$

or

𐤂 $= \Gamma_{\omega} - \Gamma_{\omega b}$

For clarity it could also be expressed:
$\Gamma_{total} = \Gamma_{\omega b} +$ 𐤂

To illustrate this idea, but without the intention of claiming that the method given below is the "right" answer to building a utility function, we constructed a simple example:

Let a policy be written for two years (t = 0 initially) guaranteeing that a $100 portfolio will grow to $105 (i.e., K = 105). The fee of $5.00 is charged outside of the policy; $2.50 at t = 0 and another $2.50 at t = 1

In this simple example let

σ = 10%
r = 2%
dividend yield = 0%.

Also let ω^b = 5% and ω' = 10% x (0 if guarantee is ITM at time t = 1, 1 if guarantee is OTM at time t = 1) so ω=.05 + .10 x (0 if guarantee is ITM, 1 otherwise). In reality this latter function will be decomposed into the rationality factor and ITM, but this example is purposefully simplified.

Further, let average annual lapse be assumed to be 10% (5% + average (0, .1)), since in this example, half of the time ω' = 10% (an up market) and half of the time ω' = 0 (a down market)

100,000 scenarios were generated. All of the cases use the same underlying paths. At time t = 1, the Black Scholes formula was used to value the 105 Put with only one year remaining. If the remaining value of the Put was less than the $2.50 fee for that period, the "rational" policyholders in the GMAB-dynamic behavior case lapse. In other words, 15% (5% + 10%) lapse. Otherwise, only 5% (5% + 0%) lapse.

The cases are:
2 year 105 Put
flat 10% lapse
dynamic lapse of 5% + (10% or 0%)
dynamic lapse shocked 1%; 5% + (11% or -1%)

Additional cases with 20% lapse or 0% lapse (which still "averages" to 10% as do the others)

FOOTNOTES:

[5] Gimmel "𐤂" comes from the Phoenician alphabet as opposed to Gimel "λ" from the Hebrew alphabet. The idea being that "𐤂" appears to be more "bent" or more convex than the Greek letter "Γ" to symbolize increased convexity.

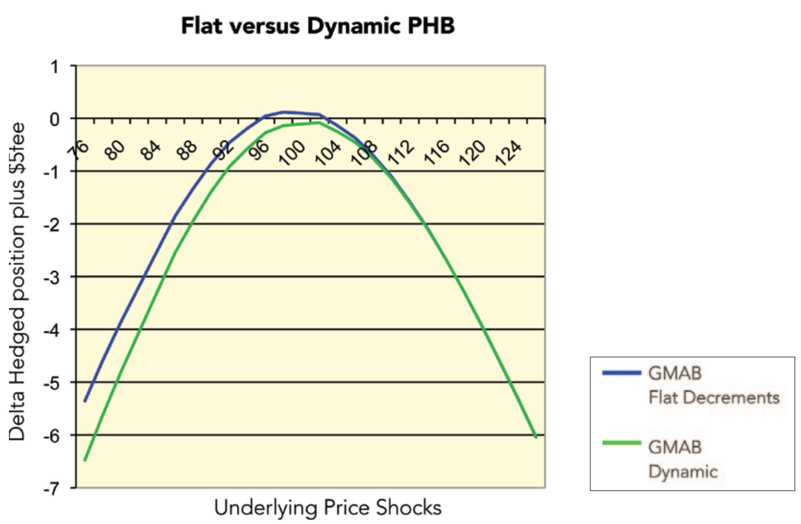
	Lapses				Survivor for Period		
	Flat		Dynamic		Flat	Dynamic	
	ωb	ω'	OTM ω	ITM ω	Sx	OTM Sx	ITM Sx
Primary Example	5%	0% to 10%	15%	5%	0.9	0.85	0.95
Example Shocked	5%	-1% to 11%	16%	4%	0.9	0.84	0.96
"Super Rational"	5%	-5% to 15%	20%	0%	0.9	0.8	1

Results:

	105 Put	GMAB-static behavior	GMAB-dynamic behavior	GMAB-shocked 1%	GMAB-super rational
Value	5.999	4.859	5.113	5.163	5.366
Delta	-50.478	-40.887	-42.897	-43.299	-44.906
Gamma	2.597	2.104	2.182	2.198	2.26
Gimmel	na	0	0.078	0.094	0.156

Gimmel does not exist for the Put itself; Gimmel is, by definition, 0 for the flat or static lapse assumption case; in the three other cases Gimmel is the difference between the Gamma of each case minus the Gamma of the flat or static lapse assumption.

The following chart shows the plotted values of a delta hedged policy shocked by price movements for both the flat lapse assumption and a dynamic lapse assumption. The blue line shows the flat lapse assumption liability, the green line shows the dynamic lapse assumption. An instantaneous movement will increase the value of the liability (more negative) when there is a dynamic assumption, hence the green line is more negatively convex than the blue line.



1) Option price plotted against stock price for base lapses => curvature = Base Gamma
2) Option price plotted against stock price for dynamic lapses => curvature =Base Gamma + Gimmel

Then the increase in curvature = Gimmel

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We hope that this term "Gimmel" and the concept it is intended to represent will help everyone have a common language in future discussions about this kind of risk. However people incorporate dynamic policyholder behavior into their models, and whatever formulae represent the policyholder utility, we hope that the common parlance is understood by practitioners so that meaningful discussions can take place without requiring that anyone disclose proprietary information about policyholder experience. ✦

APPENDIX: TAXONOMY OF OPTION SENSITIVITY METRICS

Color measures the sensitivity of the Charm, or Delta Decay to the underlying asset price. It is the third derivative of the option value, twice to the underlying asset price and once to time.

Delta measures the sensitivity of the option to changes in the price of the underlying asset.

Delta Decay, or **Charm,** measures the rate of change in the Delta of the option to the passage of time. It is the second derivative of the option value, once to price and once to time. This can be important when hedging a position over night, a weekend or a holiday.

Gamma measures the rate of change in the Delta of the option to the underlying asset.

Lambda is the percentage change in option value per change in the underlying price.

Rho measures sensitivity of the option to the applicable interest rate.

Speed measures the third order sensitivity to price. The speed is the third derivative of the value function with respect to the underlying price.

Theta measures the sensitivity of the option to the passage of time.

Vomma or Vega Gamma or Volga measures second order sensitivity to implied volatility.

Vanna measures cross-sensitivity of the option value with respect to change in the underlying price and the volatility, which can also be interpreted as the sensitivity of Delta to a unit change in volatility.

Ultima is considered as a third order derivative of the option value; once to the underlying spot price and twice to volatility.

Vega measures sensitivity to volatility.